Final questions for mathematics BSc

The final examination consists of the defence of the thesis work and answering one of the final questions below. The students may receive further short questions from related topics. The defence and answers for the questions are assessed by two separate marks.

1. **Sets, relations and functions.** Basic concepts of set theory, set operations. Ordered pairs, Cartesian product, relations. Equivalence relations and partial order. Functions. Construction of number sets.

2. **Polynomial rings.** Polynomial ring over a field. Euclidean division, greatest common divisor. Irreducible polynomials in the rings of polynomials with integer, rational, real and complex coefficients. The fundamental theorem of algebra. Partial fraction decomposition. Equations solvable by radicals. Multivariate polynomials, symmetric and elementary symmetric polynomials, the fundamental theorem of elementary symmetric polynomials. Vieta's formulas.

3. **Number theory.** Linear congruences, linear Diophantine equations. Arithmetic functions. Prime numbers and their properties. Algebraic numbers, algebraic integers. Algebraic number fields. Degree, basis, ring of integers, group of units.

4. **Linear algebra.** Vector spaces, basis, dimension. Determinants and their properties, expansion formula. Matrix operations, inverse of a matrix. Systems of linear equations, Cramer's rule. Linear transformations, eigenvalues, eigenvectors.

5. **Euclidean and unitary vector spaces.** The concept of Euclidean and unitary spaces: inner product, norm, distance, angle. The Cauchy-Schwarz inequality. Orthonormal basis, the Gram-Schmidt process, orthogonal complement. Linear forms in inner product spaces. Self-adjoint, orthogonal and normal transformations. Reduction of a quadratic form to canonical form.

6. **Algebra.** Groups, subgroups, Lagrange's theorem, permutation groups, Cayley's theorem. Direct product. Fundamental theorem of finite Abelian groups. Normal subgroups and factor groups. The fundamental theorem on homomorphisms. Rings, Euclidean rings. Problems related to constructions in geometry.

7. **Combinatorics.** Permutations, variations, combinations. Properties of binomial coefficients, the binomial theorem. Pigeonhole principle, inclusion–exclusion principle. Fundamentals of graph theory. Eulerian trail, Hamiltonian path and Hamiltonian cycle.

8. **Sequences and series of real numbers.** Convergent sequences of real numbers, monotonicity and boundedness. Limits and operations. Convergent series of real numbers. Convergence tests. Power series. Cauchy–Hadamard theorem. Elementary functions.

9. **Limit and continuity of univariate functions.** Limits of real functions. Limits and operations. Continuous real functions. Limits of functions by convergent sequences. Operations and continuity. Continuity and topology.

10. **Differential calculus of real functions.** Derivatives of univariate functions, rules of differentiation. Analysis of functions (monotonicity, extreme values, convexity). Mean value theorems in differential calculus. L'Hospital's rule.

11. **Differential calculus of functions of several variables.** Limit and continuity of functions of several variables. Partial derivative, directional derivative, total derivative. Representation of the derivative, rules of differentiation. Higher-order derivatives, Schwarz-Young theorem. Extreme values.

12. **Integral calculus.** Indefinite integral of univariate functions. Integration rules. The Riemann integral. The fundamental theorem of calculus. Improper integral. Integration over Jordan measurable sets. Fubini's theorem. Integral transforms.

13. **Differential equations.** Solution of a differential equation, and initial value problems. Global existence and uniqueness theorem, Peano existence theorem. Systems of linear differential equations with constant and nonconstant coefficients. Higher order equations. Elementary differential equations.

14. **Absolute geometry.** Axiomatic system of absolute geometry. Euclid's parallel postulate. Presentation of one model of the hyperbolic plane.

15. **Euclidean geometry.** Isometries and homotheties in the Euclidean plane and space. Area of polygons, Peano-Jordan measure in the plane. Volume.

16. **Analytic geometry.** Vectors of the Euclidean plane and space, vector addition, multiplication with scalar. Inner product, cross product. Implicit and parametric equations of lines in the plane and space. Implicit and parametric equations of planes in the space. Distance and angle of objects in the space. Geometric interpretation of conic sections, canonical implicit equations.

17. **Affine and projective geometry.** Affine and projective planes, projective completion of affine planes. Cross-ratio in the real projective plane, the invariance of the cross-ratio. Desargues' theorem and Pappus's theorem. Collineations.

18. **Differential geometry of curves and surfaces.** Differentiable curves in the plane and space. Curvature, torsion. The fundamental theorem of space curves. Surfaces in the Euclidean space. Measuring on a surface, the first fundamental form. The second fundamental form. The shape operator, principal curvatures and principal directions, Gaussian and mean curvature.

19. **Probability theory.** Kolmogorov probability space. Probability distribution, cumulative distribution function, density function. Independent variables. Expected value, deviation. Random vectors. Weak and strong laws of large numbers. The central limit theorem.

20. **Statistics.** Samples and statistics. Point estimators: unbiased and consistent estimators. Methods of estimation: method of moments, maximum likelihood estimation. Hypothesis testing. Regression analysis, analysis of variance.